

Lacroix and the Calculus

By João Caramalho Domingues. Basel (Birkhäuser). 2008. ISBN 978-3-7643-8637-5. 468 pp. US\$169.00

Silvestre¹ François Lacroix lived from 1765 to 1843. He spent most of his life in Paris, which was arguably the center of the mathematical universe during his lifetime. Although Lacroix was not one of the major research mathematicians of his generation, he was influential as a teacher and examiner, an administrator, and a textbook author. Between 1795 and 1816, he wrote more than half a dozen books. Most were elementary textbooks that went through numerous editions and were translated into a variety of languages, including English translations by the Analytical Society in Great Britain and by John Farrar in the United States. His most significant book, however, was not a textbook for students, but rather an advanced reference work for those already familiar with the calculus, the *Traité du calcul différentiel et du calcul intégral* [Lacroix, 1797–1800], published in three volumes between 1797 and 1800. Lacroix published a second edition between 1810 and 1819 and also distilled a more elementary single-volume textbook from its contents in 1802, called *Traité élémentaire de calcul différentiel et de calcul intégral* [Lacroix, 1802].

The focus of João Caramalho Domingues' book *Lacroix and the Calculus* is primarily on the three-volume *Traité*, which he refers to as the “large *Traité*”, a practice that will be followed in this review. Because Domingues is mostly concerned with the process of its composition, rather than its later influence, he concentrates primarily on the first edition. In addition, he devotes a chapter to a detailed analysis of the differences between the first and second editions of the large *Traité*, and an even longer chapter to the 1802 *Traité élémentaire*.

Lacroix and the Calculus begins with a short biographical essay on Lacroix. At 12 pages or so, it is the longest English-language biography available anywhere. Domingues observes that a more detailed biographical study is sorely needed, although his chapter is certainly more than adequate for the study at hand. Based on what Domingues has uncovered, it certainly appears that there is more than enough primary material in the various archival sources, should someone wish to address this need.

The biographical chapter is followed by a detailed overview of the large *Traité*, which begins with a volume on the differential calculus, continues with one on the integral calculus, including differential equations, and concludes with a volume on finite differences and series.² This organization is similar to Euler's series of calculus texts. Roughly speaking, Lacroix's first volume corresponds to Euler's large *Calculi differentialis* [Euler, 1755] and his second volume corresponds to Euler's three-volume *Calculi integralis* [1768–1770]. The elementary material of Euler's *Introductio in analysin infinitorum* [Euler, 1748] is represented in portions of the first volume of the large *Traité*, while some of Euler's more advanced topics are to be found in various places in the third volume. Indeed, Lacroix pointed to [Euler, 1768–1770] in a letter to Legendre of 1789 as one of the most complete

¹ Although Lacroix himself generally used only his initials, both spellings “Silvestre” and “Sylvestre” are found in manuscript sources. Whereas most modern authors seem to choose the latter spelling, Domingues prefers the spelling “Silvestre”, which was used on Lacroix's birth certificate and in his citation for the Légion d'Honneur.

² In the 1797–1800 edition, the third volume was separately titled *Traité des différences et des séries*, serving as a follow-up volume to the *Traité*, while in the second edition it was simply titled volume 3 of the *Traité*.

calculus textbooks then available. It was Lacroix's goal to write a more current book that would similarly bridge the gap between elementary texts and the memoirs in the research journals. Thus, Lacroix aimed to give a more or less complete account of a century of advances in the differential and integral calculus, from l'Hôpital's *Analyse* [l'Hôpital, 1696] onwards.

The large *Traité* is more than 1800 pages long and contains much material that has become obscure over the course of the intervening centuries. How then might one do justice to it in a book of reasonable length? Domingues chooses to organize the main portion of his narrative into five chapters, each of which explores a particular topic that is both representative of contents of the large *Traité* and likely to be of interest to historians of our time. The topics he treats from volume one are the principles of the calculus (in his Chapter 3), and analytic and differential geometry (Chapter 4). From volume two, Domingues chooses to focus on numerical integration and the concept of the integral (Chapter 5), and on types of solutions of differential equations (Chapter 6). In his Chapter 7, he surveys some of the results from volume three of the large *Traité*.

The structure is similar in each of these five central chapters. Within each topic, Domingues first surveys the major results of 18th-century mathematicians, then describes the account of the same topic given by Lacroix in the large *Traité*. In Chapter 3, for example, Domingues begins with a survey of the competing foundational notions at the end of the 18th century: infinitesimals, limits, and the power series formalism of Lagrange. He also gives an account of Carnot's work on the compensation of errors. The foundational approach that Lacroix actually adopts for his treatise is the Lagrangian one, but he also describes the alternative approaches, which he will occasionally make use of later in the large *Traité*.

After completing his review of these major topics, Domingues discusses both the 1802 *Traité élémentaire* and the second edition of the large *Traité*. This is followed by a brief discussion of the original aspects of the large *Traité*, both actual and misattributed, and the style and impact of the book. Domingues also raises some questions for further research. The book ends with a number of appendices, including some fascinating transcriptions of original documents not available elsewhere. Included are two previously unpublished memoirs, as well as syllabi from courses that Lacroix taught at the École Polytechnique.

Although many authors have written about particular aspects of the large *Traité*, this book is the first to provide a thorough study of the entire treatise. It manages to be simultaneously broad and deep in its analysis of the treatise and the process of its composition. It provides a snapshot of the calculus at the end of Euler's century, on the threshold of Cauchy's new analysis. It is an essential volume for those interested in the history of analysis and should have a place in almost any university library.

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Kelvin: Life, Labours and Legacy

Edited by Raymond Flood, Mark McCartney, and Andrew Whitaker. Oxford (Oxford University Press). 2008. Hardback. ISBN: 978-0199231256. 352 pp. £55.00.

Packaged in a sleek book jacket and printed on fine glossy paper, *Kelvin: Life, Labours and Legacy* appears from the outset to be a well-developed contribution to the historiography of Lord Kelvin, previously William Thomson (1824–1907)—a historiography that has increasingly sought, over the past two decades, to rehabilitate the reputation of a major 19th-century figure in natural philosophy, engineering and mathematics. As Brian Pippard explains in the foreword, Whig-oriented histories have all too often eclipsed Kelvin's brilliance by elevating in his place actors such as James Prescott Joule (1818–1889), Rudolph Clausius (1822–1888) and James Clerk Maxwell (1831–1879). Pippard tells the reader:

Nowadays Maxwell's researches are revered and form an essential part of every physics student's lecture course, while Kelvin's hardly get a mention. To be sure, we still have the Kelvin temperature scale, but to most physicists this is little more than a token, his pioneering ideas on thermodynamics have been overshadowed by those of his friend James Joule and his German contemporary Rudolph Clausius.

Pippard therefore wonders:

How is it that in this, and other fields where his innovations were so important, the memory of one who had been the unquestioned leader of science and technology, the versatile and prolific inventor, should fade so soon after his death? (p. v)

Presumably, one of the book's objectives is to answer that question by refocusing the light of historical analysis on Kelvin's productive and influential labour. Various contributing authors also aim to highlight the misinformed reasons for which Kelvin's reputation has so often been swept under the rug over the course of the past century. Thus, in their editorial roles, Flood, McCartney and Whitaker have attempted to produce a compendium of literature that adds to that laudable historical effort initiated two decades ago by Crosbie Smith and Norton Wise in *Energy and Empire: A Biographical Study of Lord Kelvin* [Smith and Wise, 1989], which offered a detailed and socially-aware account of Kelvin's lifetime contributions.

Indeed, in its efforts to continue debunking superficial accounts of 19th-century natural philosophy and science, *Kelvin: Life, Labours and Legacy* is a welcome addition to the history of science, particularly due to the fact that many of the contributing authors attempt to discuss Kelvin's actions, choices and scientific accounts as reasonable products of the